54241

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I SEMESTER B.Sc. EXAMINATION – MARCH/APRIL 2022
SCHEME: SEMESTER- CBCS
MATHEMATICS (PAPER – I)
ALGEBRA I AND CALCULUS I

035

Time: 03 Hours **Max Marks: 80**

Instructions to Candidates: i) Answer all five questions.

ii) Question 1 carrier 20 marks and remaining questions carry 15 marks.

Part - A

I Answer any TEN questions. **10x2=20**

a) If λ is the Eigen value of the matrix A, prove that $\frac{1}{\lambda}$ is the Eigen value of the matrix A^{-1}.

b) Find the inverse of the matrix $\begin{bmatrix} 3 & -4 \\ 2 & -1 \end{bmatrix}$ using elementary row operations.

c) Find the coefficient matrix and augmented matrix of the following equations.

$x + 2y + 2z = 1,\ 2x + y + z = 2,\ 3x + 2y - 2z = 3.$

d) Solve $3x^3 - 4x^2 + x + 88 = 0$ given that $2 + i\sqrt{7}$ is a root.

e) Form the equation whose roots are the reciprocals of the roots of the equation $3x^4 - 3x^2 + 4x - 1 = 0$

f) If α, β, γ are the roots of the equation $x^3 + 3x - 4 = 0$, find $\sum \frac{1}{\beta+\gamma-3\alpha}$

g) Find the range of values of x for which the curve $f(x) = x^3 - 18x^2 + 96x + 4$ is concave downwards.

h) Find the n^{th} derivative of $\frac{1}{2x+3}$

i) Evaluate: $\int_0^{\pi/2} sin^4x\ cos^3x dx.$

j) Find the angle between the radius vector and the tangent to the curve $r = ae^{\theta cot\alpha}$

k) Find $\frac{ds}{d\theta}$ for $x = a(1 - cos\theta), y = a(\theta + sin\theta)$

l) Find the pedal equation of the curve $r = a\theta$.

PTO

Part – B

II Answer **any THREE questions from each unit.**
(Each question carries 5 marks) **3x5=15**

a) Find the Rank of the matrix

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$$

a) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$$

c) Using Cayley – Hamilton theorem, find the inverse of the matrix.

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & 5 \end{bmatrix}$$

d) Test the consistency and hence solve.

$7x + 5y - z = 6$

$9x + 6y + 2z = -10$

$4x - 2y - z = -9$

e) Show the row equivalence of the matrices

$$\begin{bmatrix} 1 & 1 & 2 \\ -2 & -1 & -3 \\ 5 & 2 & 7 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

III **3x5=15**

a) Solve $4x^3 + 20x^2 - 23x + 6 = 0$ given that it has multiple roots.

b) Solve $x^4 - 4x^3 - 17x^2 + 24x + 36 = 0$ given that the product of two of its roots is 12.

c) Solve $x^3 - 27x + 54 = 0$ by cardon's method.

d) Solve $3x^5 - 10x^4 - 3x^3 - 3x^2 - 10x + 3 = 0$

e) Solve $x^4 - 3x^2 - 6x - 2 = 0$ by Descarte's method.

IV **3x5=15**

a) If $y = \frac{\sin h^{-1}x}{\sqrt{1+x^2}}$, prove that

$(1 + x^2)y_{n+2} + (2n + 3)xy_{n+1} + (n + 1)^2 y_n = 0$

PTO

b) Find the point of inflexion for $x = log(y/x)$

c) Find the extremum values of the function
$f(x) = 2x^3 - 3x^2 - 36x + 10$

d) Obtain the reduction formula for $\int sin^n x\, dx$ and hence evaluate $\int_0^{\pi/2} sin^n x\, dx$

e) Evaluate $\int_0^{\pi} \frac{sin^4\theta}{(1+cos\theta)^2}\, d\theta$

V

3x5=15

a) Find the angle between the curves
$r = a(1 + cos\theta)$ and $r = b(1 - cos\theta)$

b) Find the pedal equation of $y^2 = 4a(x + a)$

c) Find the radius of curvature of the curve $r^n = a^n cosn\theta$

d) Find the Centre of curvature at (4,2) of the parabola, $y^2 = 16x$

e) Find the evolute of the curve
$x = r(cos\theta + \theta sin\theta), y = r(sin\theta - \theta cos\theta)$

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