22150

II SEMESTER B.Sc. EXAMINATION, JULY/AUGUST 2023

030

SCHEME: SEMESTER (NEP)

MATHEMATICS

ALGEBRA-II (NUMBER THEORY) AND CALCULUS-II

Time: 2 ½ Hours **Max Marks: 60**

Instructions: 1. Answer all questions.
2. First question carries 12 marks and remaining questions carry equal marks.

1. **Answer any SIX questions. Each question carries two marks.** 6x2=12
   - a) Solve 16x = 8 (mod 17). CO1 LL1
   - b) Find $\emptyset(300)$. CO1 LL1
   - c) State Rolle's Theorem for the function f(x). CO2 LL1
   - d) Evaluate: $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{sinx}\right)$. CO2 LL1
   - e) If $u = x^y$ $find\ \frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$. CO3 LL1
   - f) If $f(x,y) = x^2 + y\log x$, find $f_{xy}(1, 1)$. CO3 LL1
   - g) Evaluate: $\int (2y + x^2)\ dx - (3x - y)\ dy$ along the line $x = 2t$ and $y = t^2 + 3$ where $0 \le t \le 1$. CO4 LL1
   - h) Evaluate: $\int_0^1 \int_0^1 (x + y) dy\, dx$. CO4 LL1

2. **Answer any THREE questions. Each question carries four marks.** 3x4=12
   - a) State and prove Wilson's Theorem. CO1 LL2
   - b) Find the GCD 216 & 360 and express it as a linear combination of these two integers. CO1 LL2
   - c) Solve $x \equiv 5(mod\ 7)$; $x \equiv 1(mod\ 4)$. CO1 LL2
   - d) If p is prime and (a,b) = 1 the prove that $a^{(p-1)} \equiv 1(mod\ p)$. CO1 LL2
   - e) Find the remainder when $3^{100}X2^{50}$ is divisible by 5. CO1 LL3

3. **Answer any THREE questions. Each question carries four marks.** 3x4=12
   - a) Verify Continuity of the function $f(x) = \frac{1}{1+e^x}$ at x = 0. CO2 LL2

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b) Prove that the limit of a function at a point, if it exists is unique. CO2 LL2

c) Let f(x) and g(x) be two functions such that CO2 LL2

$\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$ then prove that $\lim_{x\to a}[f(x) + g(x)] = l + m$

d) Expand log $(1 + x)$ upto the term containing x^5 by using Maclaurin's Theorem.

CO2 LL2

e) Evaluate $\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. CO2 LL2

4. Answer any THREE questions. Each question carries four marks 3x4=12

a) State and prove Euler's Theorem for homogeneous function. CO3 LL1

b) Find $\frac{dz}{dt}$. If $z = \log\ (x^2 + y^2)$ where $x = e^{-t}, y = e^{t}$. CO3 LL2

c) Find the Second Taylor Polynomial of the function $f(x, y) = \sqrt{1 + x + y^2}$ at x=1 and y=0.

CO3 LL3

d) If $x = r\cos\theta, y = r\sin\theta$, find the Jacobians J and J' and also verify $JJ' = 1$.

CO3 LL2

e) If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. CO3 LL3

5. Answer any THREE questions. Each question carries four marks 3x4=12

a) Evaluate: $\int (x + y)dx + (y - x)dy$ along the curve CO4 LL2

$x = 2t^2 + t + 1, y = t^2 + 1$ and $0 \le t \le 1$.

b) Evaluate: $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \int_0^2 \cos\theta\ r^2\ dr\ d\theta$. CO4 LL2

c) Evaluate: $\int_0^1 \int_{x^2}^{\sqrt{x}} xy(x + y)\ dy\ dx$ by changing the order of Integration. CO4 LL3

d) Evaluate: $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\ dz\ dy\ dx$. CO4 LL2

e) Find the volume of tetrahedron bounded by the planes $x = 0,\ y = 0,\ z = 0$ & $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. CO4 LL2

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