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No. of pages: 03

V SEMESTER B.Sc EXAMINATION – MARCH/APRIL 2022
SCHEME: SEMESTER – CBCS
MATHEMATICS
Real Analysis – II and Algebra – III (DSE)

038

Time: 03 Hours **Max Marks: 80**

Instructions: 1. Answer all five questions.
2. First question carries 20 marks and remaining questions carry 15 marks.

1. Answer any Ten questions. Each question carries two marks. **10x2=20**

a) Find the infimum and supremum of the sequence $\{\frac{1}{3}, \frac{4}{9}, \frac{13}{27}, \frac{40}{81}, \ldots\}$

b) Find the limit o the sequence $\{ \left(1+\frac{3}{n}\right)^{n+1} \}$

c) Show that the sequence $\{\frac{2n+3}{3n+5}\}$is monotonically increasing.

d) Find the series where nth partial sum is $\frac{n}{n+2}$

e) Test the series $\sum_{n=1}^{\infty} \frac{1}{n} \sin\frac{1}{n}$ for convergence.

f) State Raabe's test for convergence of series of positive terms.

g) In a ring R, show that $x(-y) = (-x)y \; \forall x, y \in R$.

h) Describe the quotient field of Z_5.

i) Show that $3+\sqrt{2}$ and $5+4\sqrt{2}$ are associates in $Z(\sqrt{2})$.

j) Find whether x^3+2 is reducible over Z_5.

k) Show that every homomorphic image of a commutative ring is commutative.

l) Test the reducibility of the polynomial x^2+1 over Z_3.

2. Answer any Three questions. Each question carries five marks. **3x5=15**

a) Show that the sequence $\{a_n\}$ defined by $an = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n}$ converges.

b) Show that the sequence $\{x_n\}$ given by $x_1 = \sqrt{3}$ and $x_{n+1} = \sqrt{3x_n}$ converges to 3.

c) Discuss the convergence of the following sequences.

i) $\{\frac{\log(n+1) - \log n}{\sin 1/n}\}$

ii) $\{\frac{3n^2+2n}{4n^3+n^2} \tan\left(\frac{\pi}{n^2}\right)\}$

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d) Define limit of sequence prove that the limit of a sequence is unique.

e) Define Cauchy sequence. Prove that the Cauchy sequence is convergent.

3. Answer any THREE question. Each question carries five marks. 3x5=15

a) State and prove D' Alembert's ratio test.

b) Examine the convergence of the series

$$\sum_{n=1}^{\infty}\left(\frac{n^2+n+1}{n!}\right)x^n$$

c) Test the convergence of the series

$$\sum_{n=1}^{\infty}\left(\frac{nx}{n+1}\right)^n$$

d) Discuss the convergence of

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\text{------}$$

e) Sun to infinity of the series

$$\frac{5}{3.6}+\frac{5.7}{3.6.9}+\frac{5.7.9}{3.6.9.12}+\text{------}$$

4. Answer any THREE questions. Each question carries five marks. 3x5=15

a) Prove that the set $Z[i]=\{a+ib \mid a,b \in Z\}$ forms a ring with respect to addition and multiplication.

b) Prove that the intersection of any two subring of a ring R is again a subring of R.

c) Let $M_2(z)$ be the ring of all 2x2 matrices over Z, show that $S=\left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix},\ a,b \in Z\right\}$ is a left ideal but not right ideal of $M_2(Z)$

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d) Prove that every field is an integral domain. Give an example to show the converse is not true.

e) If $\alpha = a + b\sqrt{7} \in Z(\sqrt{7})$ and $N(\alpha) = a^2 - 7b^2$ prove that $N(\alpha\beta) = N(\alpha)N(\beta)\ \forall\ \alpha, \beta \in Z(\sqrt{7})$

5. Answer any THREE question. Each question carries five marks. 3x5=15

a) Show that $f(x) = x^3 + x + 2$ is reducible over the field Z_3 and express it as a product of irreducible factors over Z_3.

b) Find the G.C.D of the polynomials $a(x) = x^2 + 1$, $b(x) = x^6 + x^3 + x + 1$ and express it as a linear combination of a(x) and b(x).

c) State and prove the fundamental theorem of homomorphism for rings.

d) Prove the kernel of an homomorphism $f : R \to R^1$ is an ideal of R.

e) If P is an integer then prove that PZ is maximal ideal of Z if and only if P is a prime.

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