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I SEMESTER BSC EXAMINATION – MARCH/APRIL – 2022

SCHEME – SEMESTER – CBCS (NEP)

018

MATHEMATICS

ALGEBRA – I AND CALCULUS – I (DSC)

Time: 2½ Hours **Max Marks: 60**

Instructions to Candidates: Answer all the questions and
All the questions carry equal marks.

I Answer any **SIX** of the following. Each Question carries two Marks: **6x2=12**

a. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

b. Find the value of d for which the following system has a non-trivial solution.
$2x - y + 2z = 0,\ 3x + y - z = 0,\ \lambda x + 2y + z = 0$

c. Show that $x^6 + 3x^2 - 5x + 1 = 0$ has at least four imaginary roots.

d. Find the quotient and the reminder obtained by dividing
$3x^3 - 4x^2 + 2x + 1$ by $x - 3$

e. Find the pedal equation of the curve $r = a\theta$.

f. Find ϕ for the curve $r = a(1 + \cos\theta)$

g) Find the n^{th} derivative of $(ax+b)^m$

h) Evaluate: $\int_0^{\pi/2} Sin^7 x\, dx$

II Answer any **THREE** of the following.
Each Question carries four Marks: **3x4=12**

a. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 3 & 2 & 6 & 7 \end{bmatrix}$$

b. Solve completely the system of equation
$x_1 + 3x_2 + 2x_3 = 0$
$2x_1 - x_2 + 3x_3 = 0$
$3x_1 - 5x_2 + 4x_3 = 0$
$x_1 + 17x_2 + 4x_3 = 0$

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c. Verify the system of equations

$x + y - 2z = 5$
$x - 2y + z = -2$
$-2x + y + z = 4$ has a solution or not.

d. Find the inverse of the matrix using elementary row operations for

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

e. Verify the Cayley – Hamilton Theorem for the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

III Answer any **THREE** of the following.
Each Question carries four Marks: **3x4=12**

a. Solve the equation $4x^4 - 4x^3 - 25x^2 + x + 6 = 0$
Given that the difference between two of the roots is unity.

b. Solve: $x^5 - 4x^4 - 5x^3 - 5x^2 - 4x + 1 = 0$.

c. Solve the equation $4x^3 + 20x^2 - 23x + 6 = 0$
Given that it has a multiple root.

d. Solve: $x^3 - 30x + 133 = 0$ by Cardon's method.

e. Solve the equation $x^4 - 2x^2 + 8x - 3 = 0$ by Descarte's Method.

IV Answer any **THREE** of the following.
Each Question carries four Marks: **3x4=12**

a. Find the angle between the curves
$r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$

b. Show that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$

c. Find the pedal equation of $x^2 + y^2 = 2ax$

d. Find the radius of curvature of $\sqrt{x} + \sqrt{y} = 1$ at (¼, ¼)

e. Find the circle of curvature of $xy = c^2$ at (c, c)

V Answer any **THREE** of the following.
Each Question carries four Marks: **3x4=12**

a. Find the n^{th} derivative of $y = \sin(ax+b)$ and hence evaluate $y = \sin(2x+3)$

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b. Find the n^{th} derivative of $y = x^2 e^{3x}$.

c. If $y = \sin^{-1} x$, then prove that $(1-x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$.

d. Prove that $\int_0^{\pi/2} cos^n\ xdx = \frac{(n-1)(n-3)\ldots\ldots\ldots}{n(n-2)\ldots\ldots\ldots} xR$

Where $R = \begin{cases} 1\ if\ n\ is\ odd \\ \pi/2\ if\ n\ is\ even \end{cases}$

e. Evaluate $\int_0^1 x^2\ (1-x^2)^{3/2}\ dx$.

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